UNSTEADY BOUNDARY LAYER FLOW AND HEAT TRANSFER IN NANOFLUIDS WITH MICROORGANISMS

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by

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LIST OF ABBREVIATIONS

IVP	Initial Value Problem
ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations
RKF45	Runge-Kutta-Fehlberg Fourth-Fifth Order Numerical Method
BVP	Boundary Value Problem

LIST OF SYMBOLS

$\bar{a}$	constant $\left(s^{-1}\right)$
$\dot{\bar{a}}$	time-dependent rate $\left(ms^{-1}\right)$
a_1	velocity slip parameter
A	unsteadiness parameter
A_1	constant $\left(s^{-1}\right)$
$\tilde{b}$	chemotaxis constant (m)
b_1	thermal slip parameter
C	nanoparticles volume fraction
c_p	specific heat at constant pressure $\left(J/kgK\right)$
c_2	viscosity parameter
c_4	thermal conductive parameter
c_6	mass diffusivity parameter
c_8	microorganism diffusivity parameter
D_B	Brownian diffusion coefficient $\left(m^2/s\right)$
D_n	variable micro-organism diffusion coefficient $\left(m^2/s\right)$
D_T	thermophoretic diffusion coefficient $\left(m^2/s\right)$
D_1	thermal slip factor (m)
d_1	micro-organism slip parameter
E_1	mass slip factor (m)
F_1	micro-organism slip factor (m)
$f(\eta)$	dimensionless stream function
$\vec{j}$	vector flux of micro-organisms $\left(kg/m^2s\right)$

k	thermal conductivity (W/mK)
Nb	Brownian motion parameter
Nt	thermophoresis parameter
p	pressure
Pe	bioconvection Péclet number
Pr	Prandtl number
$\bar{r}$	dimensional radial axis (m)
s	Stefan blowing parameter
Re	Reynolds number
L_1	velocity slip factor (s/m)
Lb	bioconvection Lewis number
Le	Lewis number
$\bar{t}$	dimensional time (s)
T	nanofluid temperature (K)
$\bar{u}, \bar{v}, \bar{w}$	velocity components along the $\bar{x}, \bar{y}, \bar{z}$ axes respectively (ms^{-1})
$\vec{v}$	velocity vector
$\hat{\vec{v}}$	average swimming velocity vector of micro-organism
$\bar{v}_w$	dimensional inflow/outflow velocity (ms^{-1})
W_c	maximum cell swimming speed (ms^{-1})
$\bar{x}, \bar{y}, \bar{z}$	dimensional cartesian coordinates (m)

Greek letters

α	effective thermal diffusivity (m^2/s)
α_1	wall expansion ratio
β	constant of the expansion/contraction strength (s^{-1})

γ	leading edge accretion/ablation
ε_1	stretching/shrinking parameter
η	similarity variable
$\theta(\eta)$	dimensionless temperature
θ_1	constant
μ	dynamic viscosity (kg/ms)
υ	kinematic viscosity (m^2/s)
ρ	fluid density (kg/m^3)
$(\rho c)_f$	volumetric heat capacity of the fluid (J/m^3K)
$(\rho c)_p$	volumetric heat capacity of the nanoparticle material (J/m^3K)
τ	ratio of the effective heat capacity of the nanoparticle material to the fluid heat capacity
$\phi(\eta)$	dimensionless nanoparticles volume fraction $(-)$
ϕ_1	constant
$\chi(\eta)$	dimensionless number of motile micro-organisms
χ_1	constant

Subscripts

$(\ )'$	ordinary differentiation with respect to η
$(\ )_w$	condition at wall
$(\ )_\infty$	condition at ambient

ALIRAN LAPISAN SEMPADAN DAN PEMINDAHAN HABA TAK MANTAP DALAM NANO BENDALIR MENGANDUNGI MIKROORGANISMA

ABSTRAK

Aliran lapisan sempadan yang melibatkan mikroorganisma telah mendapat perhatian kebelakangan ini. Beberapa masalah yang melibatkan aliran sedemikian dikaji dengan terperinci dalam tesis ini. Masalah–masalah ini melibatkan aliran dua matra laminar perolakan dan aliran tiga matra lapisan sempadan dengan pemindahan haba, jisim dan mikroorganisma di bawah pelbagai konfigurasi fizikal dengan kehadiran gelincir halaju, gelincir haba, gelincir jisim, gelincir mikroorganisma, semburan Stefan serta sempadan fluks jisim yang sifar. Bendalir dianggap sebagai Newtonian (air), likat, tak termampat dan mempunyai sifat-sifat pemindahan yang malar atau berubah. Hanya lapisan sempadan tak mantap yang direndam dalam nanobendalir dipertimbangkan. Untuk menyelesaikan masalah-masalah ini, persamaan-persamaan pembezaan separa bermatra yang menakluk aliran bendalir ditukar kepada persamaan-persamaan yang tak bermatra dengan menggunakan penjelmaan keserupaan yang sesuai. Pembolehubah-pembolehubah keserupaan digunakan untuk menurunkan persamaan-persamaan pembezaan separa kepada satu sistem persamaan-persamaan pembezaan biasa separuh tak linear. Persamaan-persamaan transformasi akan diselesaikan secara berangka menggunakan kaedah tembak kaedah Runge Kutta-Fehlberg peringkat keempat-kelima. Kesan-kesan parameter-parameter yang menakluk pada halaju, suhu, pecahan isi padu zarah-zarah nano, ketumpatan mikroorganisma-mikroorganisma mobil, pekali geseran kulit, nombor Nusselt setempat, nombor Sherwood setempat dan ketumpatan mobil untuk kadar pemindahan mikroorganisma-mikroorganisma yang tak bermatra adalah diilustrasikan secara grafik dan dalam bentuk jadual. Ia didapati bahawa parameter-

parameter kawalan mempengaruhi aliran bendalir dan ciri-ciri pemindahan haba. Kami telah membanding keputusan-keputusan berangka kami dengan keputusan-keputusan yang telah diterbitkan bagi beberapa kes-kes mengehadkan dan mendapati persetujuan yang sangat baik.

UNSTEADY BOUNDARY LAYER FLOW AND HEAT TRANSFER IN NANOFLUIDS WITH MICROORGANISMS

ABSTRACT

Boundary layer flows involving microorganism have attracted attention in recent years. Several problems involving such flows are investigated in detail in this thesis. The problems involve two dimensional laminar convective and three-dimensional boundary layer flow with heat, mass and microorganism transfer under various physical configurations in the presence of velocity slip, thermal slip, mass slip, microorganism slip, Stefan blowing and zero mass flux boundary conditions. The fluid is assumed to be Newtonian (water), viscous, incompressible and has constant or variable transport properties. Only unsteady boundary layers immersed in a nanofluid has been taken into consideration. In order to solve these problems, the dimensional partial differential equations that govern the flow are transformed into dimensionless equations by using appropriate similarity transformations. Similarity variables are used to reduce the partial differential equations to a system of nonlinear semi-coupled ordinary differential equations. The transformed equations are solved numerically using the shooting method utilizing Runge-Kutta-Fehlberg fourth-fifth order method. The effects of the governing parameters on the dimensionless velocity, temperature, nanoparticle volume fraction, density of motile microorganisms, skin friction coefficient, local Nusselt number, local Sherwood number and motile density of microorganisms transfer rate are illustrated graphically and in tabular form. It was found that the controlling parameters strongly affect the fluid flow and heat transfer characteristics. We compared our numerical results with published results for some limiting cases and found excellent agreement.

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CHAPTER 1

INTRODUCTION

The research reported in this thesis deals with unsteady boundary layer flow and heat transfer in nanofluids with microorganisms. This chapter gives a general introduction to the study. It mainly focuses on the background, research justification, problem statement and objectives of the study. In addition, an outline of the research methodology that will be used is discussed.

1.1 Research Background

Knowledge of heat transfer and related phenomena is important in various fields of science and engineering. Heat is the thermal energy that flows when there exists a temperature difference across a medium. Heat flows from high temperature sources to low temperature sinks and heat is unable to flow, by itself, from a body at lower temperature to that at a higher temperature (Cengel and Cimbala, 2014).

It is well known that the basic forms of heat transfer are conduction, convection, and radiation. Conduction and radiation are processes of heat transfer which depend only on temperature difference. For convection, the heat transfer depends not only on the temperature difference but also on the mass transport of the fluid medium. It is also a known fact that greater convection of heat transfer can be caused by faster motion of the fluid. For conduction and convection, heat transfer can take place only through a material (medium). However, for radiation, heat transfer can take place either through vacuum or through materials (Cengel and Ghajar, 2010). Convection can be further classified as natural or free convection, forced convection and mixed convection. If

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the mixing motion is induced by some external mechanism, then it is known to be heat forced convection. If the process is induced by body forces such as gravitational, centrifugal or Coriolis forces, then it is called natural convection. Mixed convection occurs when both forced and free convections mechanism act together such that neither can be neglected. For example, in atmospheric boundary-layer flows, heat exchangers, solar collectors, nuclear reactors and in electronic equipment. This thesis involves the study of forced convection.

Nanofluids are the fluids considered in the study of forced convection described in this thesis. Nanofluids are fluids that contain nano-sized particles (typically less than 100 nanometers) which exist in liquid suspension (called the base or host fluids). Nanofluids have been used as fluids of heat transfer. Nanofluids seek to obtain the highest most likely thermal properties at the smallest possible concentrations of nanoparticles by means of uniform dispersion and stable suspension of the nanoparticles in host fluids (Khullar et al., 2013). When used as coolants, for example, nanofluids are able to yield significant improvements in the thermal properties of host fluids. Nanofluids enable a more efficient, effective and uniform heat removal capability for systems requiring highly accurate temperature control at high heat fluxes. Common host fluids include water, lubricant and oil, ethylene glycol, kerosene etc. Typical nanoparticles are chemically stable metals (such as gold, copper), metal oxides (such as silica, alumina), oxide ceramics (such as Al_2O_3, CuO), metal carbides (SiC) and carbon (such as graphite, diamond) (Sarkar, 2011).

Nanofluid models are mathematical description of nanofluids. Two nanofluid models have often been used by researchers to study the behaviour of nanofluid and these are

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the Buongiorno model and Tiwari-Das model. Buongiorno (2006) identified seven possible mechanisms involved in nanofluid convection. These mechanisms are size of nanoparticle, inertia, particle agglomeration, Magnus effect (perpendicular force on object immersed within a fluid), volume fraction of the nanoparticle (response of nanoparticles to the temperature gradient), Brownian motion and thermophoresis. Brownian motion and thermophoresis were found to be very significant. Thermophoresis is known to act against temperature gradient i.e. particles from the region of higher temperature move to that of lower temperature. As opposed to thermophoresis, particles in Brownian motion move from higher concentration areas to lower concentration areas. However, the Tiwari-Das model focuses on volumetric fraction of nanoparticles. Tiwari and Das (2007) developed a model to analyse the behaviour of nanofluids considering the solid volumetric fraction. They have two main approaches in their investigation which are the two-phase and the single-phase models. The former considers the fluid and solid phases in the process of heat transfer whereas the latter considers the fluid phase and solid particles to be in thermal equilibrium and flow with similar local velocity (Tiwari and Das, 2007).

The convection that will be studied in this thesis is associated not only with nanofluids but also with the transfer of microorganisms. This transfer of microorganisms is called bioconvection. Bioconvection is resulted by the up swimming of self-propelled motile microorganisms that weigh higher than water, and it causes increase in density of the upper portion of the fluid layer (Nield and Bejan, 2013). Hence, the unstable density stratification in the fluid layer. Bioconvection control is essential for certain biological and biotechnological processes. For instance, controlling populations of plankton communities in the oceans and the separation of a vigorously swimming subpopulation

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of microorganisms in laboratory experiments (Kuznetsov, 2011). “Taxes” are the responses to the movements. We describe the more important ones. “Chemotaxis” implies that the swimming is due to chemical gradients whereas “phototaxis” means the movement along or opposite the direction of light, “gravitaxis” refers to the swimming under gravitational field, “gyrotaxis” means the swimming results from the balance between the physical torques generated by viscous drag and gravity operating on an asymmetric distribution of mass within the organism, and also “oxytaxis” which refers to swimming due to the oxygen gradient (Ghorai and Hill, 2000; Mehryan et al., 2016).

The phenomena that will studied in this thesis has various applications. For example, the intelligent manufacture of bio-nano-polymers allows drugs to be developed which achieve a “controlled release” and this has been shown to increase therapeutic influence in patients. Examples of such bio-nano-polymers are poly (lactic-co-glycolic acid), polylactic acid, chitosan, gelatin, poly hydroxy alkaonates, poly caprolactone and poly alkyl cyanoacrylate (Saranya and Radha, 2014). Moreover, bioconvection is important in bio-microsystems where it is utilized for mass transport enhancement and mixing. There are some significant issues in many micro-systems.

In this thesis, the impact of bioconvection with nanofluids on two, three-dimensional unsteady boundary layer flow past flat surface and cylinder are investigated. Behaviour of fluid flow near the vicinity of the surface on velocity, temperature, nanoparticle volume fraction profiles, heat, mass and the transfer rates of microorganism are thoroughly analysed according to each problem that are discussed in the corresponding chapters. The effect of bioconvection parameters and nanofluid parameters are

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investigated due to their effects on the properties of stability and rheology, significantly higher thermal conductivities with the advantage of negligible drawback in the drop of pressure (Daungthongsuk and Wongwises, 2007).

1.2 Problem Statement

Progress in scaling down devices in medical engineering requires a deep and refined understanding of fluid dynamics, heat and mass transfer phenomena at small scales. Hence, scientists and engineers are actively studying methods for enhancing heat, mass and momentum transfer strategies at the micro-dimensional scale. Devices at such scales arise in certain areas of health technology including efficient micro-sized cooling systems for biomedical processing systems, bio-electronic devices, lab-on-a-chip biological designs etc. (Hedayati and Domairry, 2016). One aspect distinguishing microscale flows from their macroscale counterparts is that the wall slip effect usually becomes so vital that its neglect may lead to predictions which differ from reality. Employing the Navier–Stokes equations together with adequate slip velocity boundary conditions has been demonstrated to be a appropriate for providing accurate results for micro/nanoscale gas flows as described by Wu (2016). Numerous studies have shown the important influence of slip velocity boundary conditions on near-wall flow characteristics (Hayat et al., 2014; Mahapatra and Nandy, 2013; Mishra and Singh, 2014; Zohra et al., 2017). For bioconvection, which is the focus of this thesis, slip velocity boundary conditions is also an important influence. An efficient bioconvection process is achieved by the careful suspension of microorganisms in nanofluids which serves to enhance thermal conductivity as well as stability of the resulting bio-nanofluid. The problem discussed in this thesis are motivated by the need to refine and improve the mathematical modelling of bioconvection nanofluid flows

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as, incorporating appropriate boundary conditions to obtain a deeper understanding of bioconvection.

It is clearly important to be able to mathematically model and understand phenomena related to bioconvection for various scientific applications and for various situations. A failure to develop robust models may hinder in-depth understanding. Further, it is important to formulate models which incorporate unsteadiness as certain situation are inherently unsteady. This research focuses on studying on the effects of unsteady boundary layer flow over sheet, plate and cylinder in nanofluids containing microorganisms. In particular, on the problems of:

(i) Unsteady stagnation point flow of bionanofluid with stretching shrinking effects: decelerating flow.
(ii) Unsteady three-dimensional stagnation point flow of bionanofluid with variables properties: stretching case.
(iii) Influence of Stefan blowing on water-based nanofluid flow submerged in microorganisms with leading edge accretion or ablation: first solutions.
(iv) Numerical study of slip effects on asymmetric bioconvective nanofluid flow in a porous microchannel with an expanding/ contracting upper wall using Buongiorno’s model.
(v) Passively controlled nanofluid slip flow with Lewis and Péclet number effects: accelerating/decelerating cylinder.

The effects are observed based on dimensionless velocity, temperature, nanoparticle volume fraction (concentration), density of motile microorganism profiles as well as

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skin friction coefficient (surface drag), Nusselt number (heat transfer rate), Sherwood number (mass transfer rate) and microorganism transfer rate. The effects of selected governing parameters are also investigated theoretically.

1.3 Objectives

The objectives of this thesis are to:

1. Formulate new mathematical models based on the equations for conservation of mass, momentum (Navier-stokes), species (nanoparticle volume fraction and microorganism) and energy for unsteady convective boundary layer flow of Newtonian nanofluid with microorganism.

2. Compute and study the solutions of the new mathematical models mentioned. In particular the effects of pertinent parameters on the quantities of engineering interest and fluid flow. This will facilitate greater understanding of bioconvective flow in Newtonian nanofluids. The parameters studied are:

   i) Suction/injection, Brownian motion, thermophoresis, slips, Lewis number, bioconvection Lewis number, Peclet number, stretching/shrinking, Reynolds number, unsteadiness, transport variable properties parameters.
   ii) The nanofluid parameters: Brownian motion, *Nb*, thermophoresis, *Nt* and Lewis number, *Le.*
   iii) The microorganism parameters: bioconvection Lewis and Peclet numbers.

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1.4 Scope and Limitation of Research

The scope of study is limited to problems involving laminar forced convective boundary layer equations for unsteady, two-dimensional/three dimensional, incompressible flow containing nanofluid and microorganisms. The nanoparticles suspension is considered to be stable and dilute and without agglomeration of nanoparticles. Increasing concentration of nanoparticles causes instability.

Further, it will be assumed that the nanoparticles have no influence on the direction and velocity of micro-organism's swimming. It is assumed that both the base fluid and nanoparticles locally are in thermal equilibrium state. The motile microorganisms, nanoparticles and base fluid are assumed to have the same velocity. These assumptions have also been made by other researchers. Further, the shape of the nanoparticles is also ignored. In this study, we use Buongiorno's model of nanofluid.

In addition, for the problems in this thesis, we consider the first solutions of the system of ODEs. First solution is stable and physically realizable while the second solution is unstable and further second solutions are physically not realizable in the fluid mechanics area (Bachok et al., 2010 (page: 4075); Nandy, 2015 (page: 23); Roşca et al., 2016 (page: 43); Sharma et al., 2014 (page: 94) ; Weidman et al., 2006 (page: 730)). Moreover, since many papers postulate that the second solution is an unstable solution and may be unrealistic, it is deemed not necessary to find and discuss in detail the multiple solution for this research. It might give a contribution in mathematics but does not give any physical meaning regarding the models (Bachok et al., 2010).

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Furthermore, it should be noted that Ghosh et al. (2016) mentioned that dual solution exist for shrinking together with suction. This situation only arises in Chapter 3 of this thesis and not Chapter 4 – 7 of this thesis. Although it was mentioned earlier that the dual solution will not be discussed in detail, the dual solution for the case of chapter 3 was found and discussed briefly in the appendix C and C1.

1.5 Methodology

The mathematical models for the physical problems considered in this thesis are based on the equations of conservation of mass, momentum, energy, nanoparticle volume fraction and density of motile microorganism. The models are expressed using a system of partial differential equations system (PDEs) together with supplementary conditions. This system of PDEs together with the supplementary condition are complicated and it is not possible to obtain exact analytical solutions. To address this problem, similarity transformation variables are used to reduce the number of independent variables to a single dependent variable and this then leads a system of ordinary differential equations (ODEs) with supplementary conditions which form a boundary value problem. Being a system of ODEs rather than PDEs makes the equations easier to solve. Even so the system is still complicated and needs to be solved numerically.

The numerical solution of the boundary value problem involving a system of ODEs resulting from the above mentioned similarity transformation by the use of a MAPLE solver (“dsolve”) will now be discussed. The boundary value problem is first changed by “dsolve” into a system of initial value problems. The precise form of this system of initial value problem is determined by the use of the shooting method (Burden and

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Faires, 2011) This associated initial value problem is solved numerically using the Runge-Kutta-Felhberg fourth-fifth (RKF45) method. The accuracy and robustness of RKF45 has been well established in several studies (for example, Butcher, 1987; Mathews and Fink, 2004; Aziz et al., 2012a; Uddin et al., 2013). The results obtained using "dsolve" are plotted in figures and tables which are then analyzed.

Actually, any standard numerical procedure for solving an initial value problem involving ODEs such as Euler, the classical RK4, Adam Bashforth can be used but dsolve uses RKF45 so that the error is consistently less than a predetermined value. Hence the error is controlled.

Let y_i denotes the approximation to $y(x_i)$ i.e. the value of the solution at the *i*-th time level and let *h* denote the time step. For RKF45, at the *i*-th time level, the values of K_1 till K_6 are computed as follows:(Burden and Faires, 2011)

$$\begin{aligned}
K_1 &= h\,f(x_i, y_i),\\
K_2 &= h\,f(x_i + \frac{1}{4}h, y_i + \frac{1}{4}K_1),\\
K_3 &= h\,f(x_i + \frac{3}{8}h, y_i + \frac{3}{32}K_1 + \frac{9}{32}K_2),\\
K_4 &= h\,f(x_i + \frac{12}{13}h, y_i + \frac{1932}{2197}K_1 - \frac{7200}{2197}K_2 + \frac{7296}{2197}K_3),\\
K_5 &= h\,f(x_i + h, y_i + \frac{439}{216}K_1 - 8K_2 + \frac{3860}{513}K_3 - \frac{845}{4104}K_4),\\
K_6 &= h\,f(x_i + \frac{1}{2}h, y_i - \frac{8}{27}K_1 + 2K_2 - \frac{3544}{2565}K_3 + \frac{1859}{4104}K_4 - \frac{11}{40}K_5).
\end{aligned} \tag{1.1}$$

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Then an approximation to the solution of IVP is made using a Runge-Kutta method of order 5

$$\tilde{y}_{i+1} = y_i + \frac{16}{135}K_1 + \frac{6656}{12825}K_3 + \frac{28561}{56430}K_4 - \frac{9}{50}K_5 + \frac{2}{55}K_6 \qquad (1.2)$$

and another solution is determined by using a Runge-Kutta method of order 4:

$$y_{i+1} = y_i + \frac{25}{216}K_1 + \frac{1408}{2565}K_3 + \frac{2197}{4104}K_4 - \frac{1}{5}K_5, \qquad (1.3)$$

Then q is determined by

$$q = \left(\frac{\varepsilon h}{2\left|\tilde{y}_{i+1} - y_{i+1}\right|} \right)^{0.25} \approx 0.84 \left(\frac{\varepsilon h}{\left|\tilde{y}_{i+1} - y_{i+1}\right|} \right)^{0.25}, \qquad (1.4)$$

where ε is the predetermined error tolerance. According to Burden and Faires (2011),

- If $q < 1$, reject the initial choice of h at the ith step and repeat the calculations using qh, and
- If $q \geq 1$, accept the computed value at the ith using the step size h and change the step size to qh for the $(i + 1)$st step.

Note that, another popular software for solving BVPs problem is the MATLAB solver bvp4c function which is based on finite difference method. In this thesis, the Maple solver “dsolve” was used because it is often used in heat transfer studies and is reasonably straight forward to use. Comparison of the solutions obtained from Maple

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“dsolve” with solutions from the Matlab “bvp4c” indicate good agreement and supports the validity and the accuracy of our numerical computations.

The solutions on the dimensionless flow and heat, mass and microorganism transfer rates are investigated and presented in figures and tables in each problem formulated in Chapters 3-7.

1.6 Thesis Organization

The organization of the thesis is as follows: Chapter 1 discusses important information regarding the work in the thesis such as research background, problem statement, research objective, scope of the study and research methodology in accordance. Furthermore, Chapter 2 discusses the significant literature related to the area of research and significance for this research are reviewed. The review is divided into several sections based on the types of problems associated such as stagnation point flow, nanofluid model, bioconvection, nanofluid with bioconvection and finally boundary conditions. Moreover, the basic governing equations in vector form are included in this chapter.

Chapter 3 studies the governing equations and boundary conditions for the problem of unsteady on bioconvective flow of nanofluid over a sheet with two-dimensional stagnation point flow. The governing equations for the problem of unsteady forced convective three-dimensional stagnation point flow past a sheet using variable transport properties containing microorganism immersed in a nanofluid are discussed in Chapter 4. Chapter 5 is on unsteady boundary layer flow past a flat surface in a nanofluid bioconvection considering Stefan blowing effects. Chapter 6 discusses the problems which “slips” takes into account rather than “no slip” boundary conditions

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for the parallel plate flow. Chapter 7 reveals boundary layer bionanofluid slip flow past a solid horizontal cylinder. Finally, Chapter 8 contains the conclusion of this study. In addition, this chapter also discusses possible further works in this field of research.

CHAPTER 2

LITERATURE REVIEW AND BASIC EQUATIONS

2.1 Introduction

A literature review serves as an overview to an in-depth academic study. From a literature review one can identify gaps in current knowledge or issues to be resolved. In this chapter, a literature review of previous research relevant to this thesis as well as background knowledge necessary is presented.

2.2 On Stagnation Point Flow

Stagnation point fluid flows are essentially the flows of fluid past a surface in which pressure is maximum (Wang and Ng, 2013). According to Mabood and Khan (2014), application of stagnation point flow arises in thrust bearings and diffusers designs, transpiration cooling and oil recovery as well as many other situations. Two-dimensional stagnation flows were first studied by Hiemenz more than a century ago and since then many aspects of stagnation problem have been researched.

Wang (2008) observed that exact similar solutions of the Navier-Stokes equations are able to define the characteristics of stagnation flows. Mustafa et al. (2011) studied two dimensional flows of nanofluids towards a stretching sheet. The characteristics of flow and heat transfer of three-dimensional stagnation point flows using Tiwari and Dass nanofluid model was examined by Bachok et al. (2010). Roldan-Alzate (2008) also studied stagnation point flows in the physiological flows context. The streamlines show the stagnation region. The importance of researches on stagnation point flows in physiology was emphasized by Ambrosi et al. (2012) and Humphrey and Rourke (2015).

Ibrahim et al. (2013) studied the effect of magnetic field on stagnation point flow and heat transfer caused by nanofluid flow towards a stretching sheet. In addition, Nandy and Mahapatra (2013) studied velocity slip and heat generation/absorption effects on magnetohydrodynamic (MHD) stagnation-point flow and heat transfer over a surface that stretches/shrinks. They included conditions of convective boundary in the presence of nanoparticle volume fractions.

In all of the above studies the stagnation point flow arises when the line of stagnation is perpendicular to the stretching surface. However an oblique line (non-orthogonal) stagnation point is important in certain situations (Ibrahim et al., 2013). Lok et al. (2006) and Abbas et al. (2017) considered non-orthogonal stagnation point flow towards a stretching sheet using the Keller-box numerical method. Both studies proved that obliqueness of a free stream line causes the shifting of the stagnation point towards the incoming flow.

A point to be noted is that the approximate numerical solutions for the unsteady stagnation point flow over stretching sheet with bioconvection has never been reported in any of the literature. Therefore, a numerical study on the flow of stagnation point past a stretching sheet with constant transport properties is presented in Chapter 3 while for variable transport properties with stretching/shrinking case this is discussed in Chapter 4.

2.3 On Nanofluid Flow

The fluid considered in this study are nanofluids whose base fluid is water. This section discusses aspects of nanofluid flow related to this study.

A significant development in materials science and thermal engineering in the past two decades has been the development of nanofluids. Nanofluids constitute a liquid suspension containing very fine particles (diameter less than 100 nm) in a base fluid like water, oil, ethylene glycol etc. Nanoparticles can be made from nitride ceramics (AlN, SiN), metals (Cu, Ag, Au) and semiconductors (SiC). Nanoparticles shapes may be disks, spheres, cylindrical rods etc. (Shehzad et al., 2016). The accumulation of nanoparticles into the base fluid can enhance the fluid flow and heat transfer proficiency of the liquids and boost the low thermal conductivity of the base fluid. Representative works on convective boundary layer flow and application of nanofluids were conducted by Buongiorno (2006), Das et al. (2007), Kakaç and Pramuanjaroenkij (2009), Wen et al. (2009) and Saidur et al. (2011). Further representative studies have been communicated by Haddad et al. (2012), Loganathan et al. (2013), Mahian et al. (2013), Nield and Bejan (2013), Rahman et al. (2014), Hatami et al. (2014), Mejri et al. (2014), Sheremet and Pop (2015), Mohyud-Din et al. (2015), Ghanbarpour et al (2015), Li et al. (2015), Vanaki et al. (2016), Zhao et al. (2016), Serna (2016), Akilu et al. (2016), Ilhan et al. (2016), Rahman et al. (2016), Sheremet et al. (2016) and Agarwal et al. (2017), amongst others.

We discuss certain studies involving nanofluids in greater detail. Ferdows et al. (2014) investigated radiative magnetohydrodynamic nano-polymer stretching flows. The study can be applied in high-temperature nano-technological materials processing.

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Uddin et al. (2015) studied numerically the stretching fluid dynamics of magnetic nano-bio-polymers. The heat exchanger may be enhanced by adding nanoparticles with high thermal properties in low volume fraction within the liquid. Conventional heat transfer fluids, for instance, water, oil and ethylene glycol are known to be weak heat transfer fluids (Loganathan and Vimala, 2014). Since the thermal conductivity of these liquids plays a major role in the heat transfer coefficient, various strategies have been explored to upgrade the thermal conductivity of these liquids. Nanomaterials have been successfully used small scale/nano electromechanical devices, advanced cooling frameworks, extensive scale thermal frameworks in evaporators, heat exchangers and mechanical cooling applications. Nanofluids are stable under a variety of operation conditions i.e. with no disintegration, sedimentation, clogging, coagulation or extra weight drop. This is because of the small size and low volume nano-particles required for thermal conductivity improvement (Hayat at al., 2016).

There are two types of model for nanofluids which have been commonly used by the researchers, namely Buongiorno's model (2006) and the Tiwari-Das model (2007). According to Buongiorno, the velocity of the nanofluid is considered as the total of both the velocities of base fluid and the relative/slip. His model emphasizes the dominant mechanisms as Brownian diffusion and thermophoresis. In contrast to Buongiorno's model, the Tiwari-Das model considers the solid volume fraction of the nanoparticles. The Buongiorno model emphasized that the Brownian diffusion and thermophoresis are the most prominent parameters the characteristics nanofluid flows.

In this thesis, we utilize Buongiorno's model of nanofluid for which Brownian motion and thermophoresis are the important parameters in dealing with bioconvection fluid flows.

2.4 On Bioconvection

Hillesdon and Pedley (1996) defined bioconvection as a formation of pattern in suspensions of microorganisms, caused by up-swimming of the microorganisms. These microorganisms (e.g. bacteria, algae) may include gravitaxis, gyrotaxis or oxytaxis organisms. Bioconvection is an essential use in biological systems and biotechnology as various aspects of bioconvection has been explored.

A precise discussion of bioconvection in suspensions of oxytactic bacteria is provided by Hillesdon and Pedley (1996). Kuznetsov and Avramenko (2003) studied how the stability of bioconvection affect the particles. Kuznetsov et al. (2004) conducted a theoretical investigation of a falling bioconvection plume in a deep chamber containing fluid saturated porous medium. Geng and Kuznetsov (2005) studied the effect of dilute suspension containing gyrotactic microorganisms has on small solid particles. The concept of effective diffusivity was introduced to study how small solid particles are affected by bioconvection.

In 2011, Kuznetsov, proposed a novel type of nanofluid that is filled with nanoparticles and motile (oxytactic) microorganisms. He showed that an addition of motile microorganisms to the suspension increases mass transfer and microscale mixing while improving the stability of nanofluid. According to his investigation, the oscillatory mode of nanofluid bioconvection is possible to be stimulated with the interaction of three competing agencies: oxytactic microorganisms, heating or cooling from the bottom, and top or bottom-heavy nanoparticle distribution.

The Galerkin method was used to get an approximate analytical solution to the problem. This work of Kuznetsov has generated interest in obtaining further understanding of bioconvection in nanofluids. Kuznetsov (2012) developed a theoretical model of the onset of convection instability stimulated by simultaneous effects yielded by oxytactic microorganisms, nanoparticles, and temperature variation. The mechanisms responsible for the slip velocity between the nanoparticles and the base fluid like the Brownian motion and thermophoresis were taken into account in the model.

Seng et al. (2012) studied the steady mixed convection boundary layer flow near the lower stagnation point of a horizontal circular cylinder. The cylinder was maintained at a constant surface temperature and was placed in a porous medium saturated by a nanofluid containing gyrotactic microorganisms in a stream flowing in an upward manner. The system of nonlinear ordinary differential equations was solved numerically with the Keller box method**.** The steady boundary layer free convection flow past a horizontal flat plate in a porous medium containing a water-based Newtonian nanofluid with gyrotactic microorganisms was explored by Aziz et al. (2012b). Similarity transformations available in the literature to modify the governing partial differential equations to similarity differential equation are used by them.

Adding microorganisms to base fluids (e.g. water) creates the process of bioconvection which is directionally-orientated swimming typically towards an imposed or naturally present stimulus e.g. light, gravity, magnetic field and chemical concentration (oxygen). The density of the microorganism is inclined to be more than the density of the free stream fluid. This can cause an instability in the density profile with subsequent upending of the fluid against gravity (Raees et al., 2015).

The base fluid has to be water for the majority of microorganisms to survive and be active and it is assumed nanoparticle suspension stays stable and do not agglomerate for a couple of weeks (Anoop et al., 2009). For bioconvection to happen, the suspension must be dilute since nanoparticles would increase the suspension's viscosity and viscosity tends to dominates bioconvection instability (Hwang and Pedley, 2014). Aziz et al. (2012a) made a theoretical study on the natural bio-convection boundary layer flow of nanofluids and verified that the bioconvection parameters affect mass, heat, and motile microorganism transport rates. Whilst Latiff et al. (2015) studied unsteady forced bioconvection slip flow of a micropolar nanofluid from a stretching/shrinking sheet.

Bioconvection plays a role to play in bio-microsystems for mass transport augmentation and microfluidic devices such as bacteria-powered micromixers (Tham et al., 2013). Other significant applications of nanofluid bioconvection arise in the synthesis of novel pharmacological agents (drugs) as elaborated by Saranya and Radha (2014) and earlier for nano-bio-gels as discussed by Oh et al. (2009). Microorganisms used to enhance biodegradable polymeric nanomaterials and improve desirable medical characteristics such as bioavailability, biocompatibility, encapsulation, DNA embedding in gene therapy, protein deliverability etc.

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2.5 On Nanofluid Bioconvection

Nanofluid bioconvection occurs when the spontaneous pattern formation and density stratification is the result of simultaneous interactions of the denser self-propelled microorganisms, nanoparticles, and buoyancy forces (Mutuku and Makinde, 2014). As we highlighted in Chapter 1, microorganisms are known to respond to certain stimuli by swimming in particular directions. These responses are called taxes. The examples would be gravitaxis, gyrotaxis, phototaxis, magneto-taxis and chemotaxis (Siddiqa et al., 2016). Gravitaxis is the swim opposing gravity and gyrotaxis is the swim that determines the equilibrium of torques due to viscous forces from shear flows and gravity. Phototaxis is the swim due to movement towards or away from light (Acharya et al., 2016). Oxytactic microorganism used for hydrodynamic convection leads to a flow system which transports cells and oxygen from the posterior fluid region to the inferior fluid regions (Deleuze et al., 2016).

Fundamentally, the swimming of microorganisms that causes the fluid to convect increases the density of the surrounding fluid. Makinde and Animasaun (2016) reported heat and mass transfer behaviour decreases the diffusion of motile microorganisms. Akbar and Khan (2016) investigated the effects of magneto-bioconvection, Brownian motion and thermophoresis on the flow of free convection over a stretching sheet.

Amirsom et al. (2016) analyzed the three-dimensional stagnation point flow of fluid filled with both nanoparticles and gyrotactic micro-organisms with variable transport properties. Babu and Sandeep (2016) simulated the non-aligned bioconvective stagnation point flow of a nanofluid comprising of gyrotactic micro-organisms from a stretching sheet by considering nonlinear radiation and variable viscosity for both instances of oblique and free stream flow.

The assumptions of this thesis are the nanoparticles do not agglomerate and its suspension remains stable. Its presence has no effect on the direction of microorganisms' swimming and their swimming velocity. Raees et al. (2015) studied the unsteady flow of fluid with nanoparticles and motile gyrotactic microorganisms between two parallel plates while keeping one moving and other fixed. Raees et al. (2016) studied three-dimensional stagnation flow of nanofluid filled with nanoparticles and microorganisms on a moving surface using anisotropic slip.

In Chapters 3 until 7 an approximation numerical solution for boundary layer flow of heat, mass, microorganism transfer over plate, sheet and cylindrical surface in a Newtonian fluid are presented. All of these respective chapters discuss Buogiorno's nanofluid with microorganism bioconvection. Note that bioconvection without nanoparticles with base fluids is not applicable in the industry.

2.6 On Boundary Conditions

Boundary conditions are very essential in simulating flows of convective nanofluid. The blowing effect comes from the species transfer of the Stefan problem. It is known that paper drying process involves massive amounts of species being transferred by evaporation (Nellis and Klein, 2008). The species diffusion yields a bulk motion of fluid and this further adds motion to the fluid (Lienhard and Lienhard, 2005).

It should be noted that this blowing effect is not the same as the mass injection or blowing where the surface is treated as porous. However, those effects considered in this thesis are for solid surfaces and the blowing is caused by transfer flux of certain species from the surface to the outside of the boundary layer. Species transfer varies according to the flow field and is affected by the mass blowing at the wall (Fang and Jing, 2014).

Fang and Jing (2014) studied the boundary layer flow considering the Stefan blowing effects and verified that the blowing velocity was proportional to the mass transfer flux. Stefan blowing and the effects of multiple-slip on buoyancy-driven bioconvection nanofluid flow with microorganisms was examined by Uddin et al. (2016). They investigated the effect of velocity slip, thermal and mass slip boundary conditions on the nanofluid bioconvective boundary layer flow over an upward facing moving horizontal plate including the Stefan blowing effects at the wall. Blowing (injection) is the introduction of fluid through a porous bounding wall that can be fully used to inject reactants, cool the surface, mitigate corrosion or scale and also modify drag (Uddin et al., 2016).

Reddy and Reddy (2013) studied the chemical reaction effects and heat generation on MHD boundary layer flow of a vertical plate with suction and dissipation in motion. Researches on the slips effect on fluids flow in different geometries and applications has been done by Cao and Baker (2009), Zheng et al. (2013), Adesanya (2014), Azese (2015), Javed et al. (2016), Zaimi and Ishak (2016) and many other researchers. It is known that in certain designs of hydrophobic fuel cell, velocity, thermal and mass slip phenomena do exist.

Khan et al. (2014b) researched on the behavior of water suspension that is filled with nanoparticles and motile gyrotactic microorganisms with magnetic field and Navier slip over a vertical plate. Second order velocity slip was investigated by Fang et al. (2010). They considered steady, two-dimensional laminar flow over a continuously shrinking sheet in a quiescent fluid.

Based on Mahapatra and Nandy (2013), fluids exhibiting slip are significant in technological applications like the polishing of artificial heart valves. With a wall boundary slip, the flow behaviour and the shear stress in the fluid differs a little bit from those in the no-slip flows. Rahman et al. (2014) researched on the flow under slip conditions over a permeable shrinking surface which can be solved numerically using MATLAB software and they reported that the velocity slip at the shrinking surface has a lot of effects on the velocity distribution and drag forces on the wall. Recently, Mahmood et al. (2016) researched on the effect of time dependent velocity slip over a lubricated rotating disk. They found that, the components of radial and tangential shear stress augment by enlarging slip parameter and decrease by enhancing unsteadiness. However tangential shear stress is an increasing function of unsteadiness without the slip parameter.

The approximate numerical treatment for the unsteady flow of nanofluid flow and microorganism transfer with Stefan blowing has never been reported in the literature. Therefore, a theoretical study on constant free stream on nanofluid flow pass on vertical surface with impermeable suction/injection is presented in Chapter 5. For slips effect on nanofluid bioconvection flow have been discussed in the Chapter 6 and 7.